

# FOM and errors from g4hrs histograms

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## Figure of merit definition

Figure of merit:

$$\text{FOM} = R \times A^2 \times S^2$$

Asymmetry error:

$$\frac{\delta A}{A} = \sqrt{\sigma_{stat}^2 + \sigma_{sys}^2}, \text{ where } \sigma_{stat} \propto 1/\sqrt{R}$$

Radius error:

$$\frac{\delta R}{R} = \frac{\delta A}{A} \left( \frac{1}{S} \right) \times 0.01$$

## Acceptance function method

Use simulation output to generate acceptance function:

$$a(\theta) = \frac{\Delta\phi}{2\pi} \left( \frac{\text{accepted } \theta \text{ distribution}}{\text{generated } \theta \text{ distribution}} \right)$$

Find accepted rate as a function of angle:

$$R(\theta) = \rho t I \left( \frac{N_A}{A} \right) \left( \frac{d\sigma}{d\Omega} \right) a(\theta)$$

Average rate  $R$  and rate-averaged  $A$ ,  $S$  calculated by:

$$R = \int R(\theta) d\Omega$$

$$A = \frac{1}{R} \int A(\theta) R(\theta) d\Omega$$

$$S = \frac{1}{R} \int S(\theta) R(\theta) d\Omega$$

where  $d\sigma/d\Omega$ ,  $A(\theta)$ ,  $S(\theta)$  are looked up in table

## Histogram method

Fill histograms from rate, asymmetry, and sensitivity for each event

Loop over all events  $i = 0 \dots N$ :

```
rateHistogram->Fill( $R_i * V_i * N$ );  
asymHistogram->Fill( $A_i$ ,  $R_i * V_i$ );  
sensHistogram->Fill( $S_i$ ,  $R_i * V_i$ );
```

where  $V$  is the phase-space volume correction discussed last week

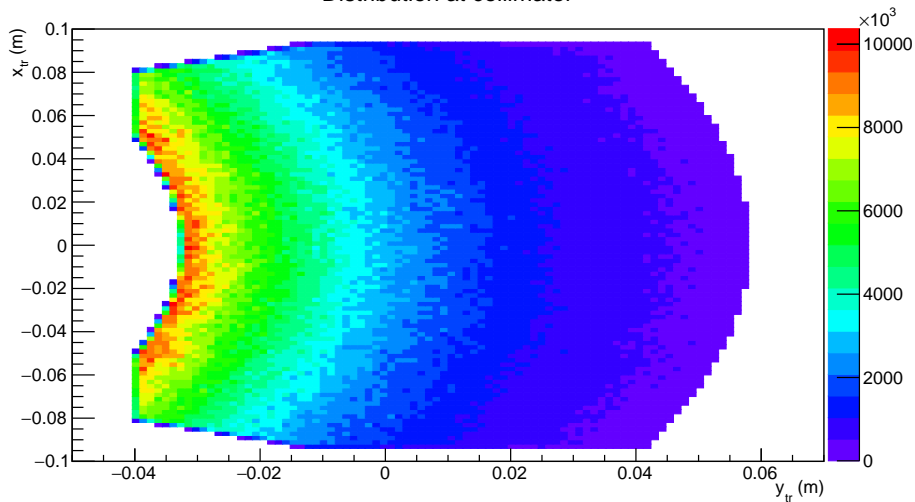
Average rate  $R$  and rate-averaged  $A$ ,  $S$  obtained  
directly from resulting distributions

Compare results of each method using g4hrs output

- $10^7$  simulated events
- Cut on nominal PREX-II collimator design (untuned  $\Delta r$ )  
→ Details on next slide
- Momentum cut  $p > 1$  GeV
- *No cut* at detector/focal plane  
→ All electrons entering Q1 counted in acceptance

# Collimator cut

Distribution at collimator



## Results: acceptance function

Results for acceptance function method at  $E = 1.063$  GeV:

$$R = 2.34 \text{ Hz}$$

$$A = 0.636 \text{ ppm}$$

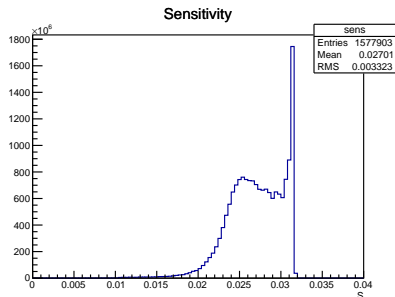
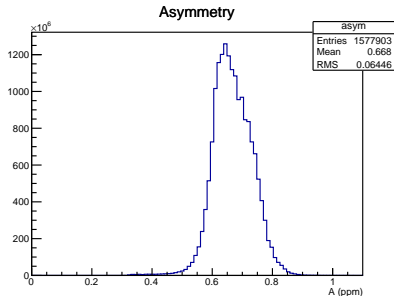
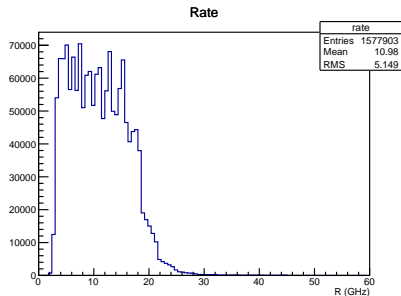
$$S = 0.0255$$

$$\text{FOM} = 6.17 \times 10^{-3}$$

$$\frac{\delta A}{A} = 0.0275$$

$$\frac{\delta R}{R} = 0.0108$$

# Results: histogram



**R = 10.975698 GHz**  
**A = 0.668025 ppm**  
**S = 0.027007**  
**FOM = 0.003573**  
**dA/A = 0.021664**  
**dR/R = 0.008022**



## Comparison

Quantity	Accept. Func.	Histogram	% difference
Rate (GHz)	2.34	10.98	369
Asymmetry (ppm)	0.636	0.668	5.03
Sensitivity	0.0255	0.0270	5.88
$\delta A/A$	0.0275	0.0217	21.1
$\delta R/R$	0.0108	0.008	25.9

# Momentum transfer

## Momentum transfer squared

