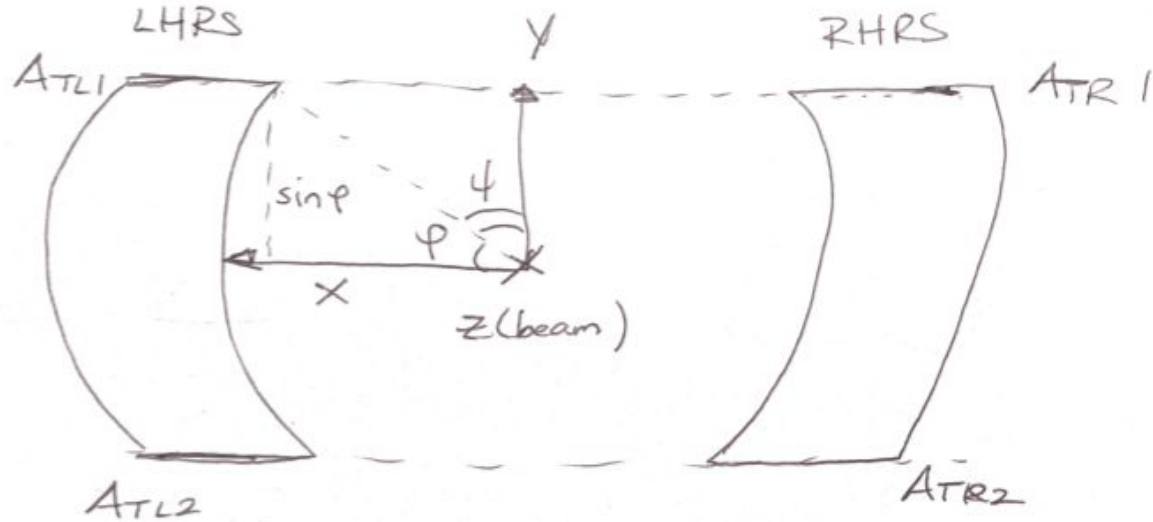


# Intro to $A_T$ Corrections

# $A_T$ Detector Picture



$$\psi + \phi = \frac{\pi}{2}, \quad \psi - \text{relative to vertical}$$

Four  $A_T$  detectors to monitor transverse (horizontal) polarization (Up/Down)

# Azimuthal Angle

Vertical  $A_T \propto \cos \varphi$  Horizontal  $A_T \propto \sin \varphi$

$\varphi$  is azimuth relative to horizontal,  $\Psi$  is azimuth relative to vertical

$$\cos(\Psi) = \sin(\varphi) \equiv \frac{\pm \theta_{tg}}{\sqrt{\theta_{tg}^2 + \sin^2 \theta_o \pm 2\phi_{tg} \tan \theta_o + \sec^2 \theta_o \phi_{tg}^2}}$$

+ sign for LHRS, - sign for RHRS

$$\sin(\Psi) = \cos(\varphi) \equiv \frac{\sqrt{\sin^2 \theta_o \pm 2\phi_{tg} \tan \theta_o + \sec^2 \theta_o \phi_{tg}^2}}{\sqrt{\theta_{tg}^2 + \sin^2 \theta_o \pm 2\phi_{tg} \tan \theta_o + \sec^2 \theta_o \phi_{tg}^2}}$$

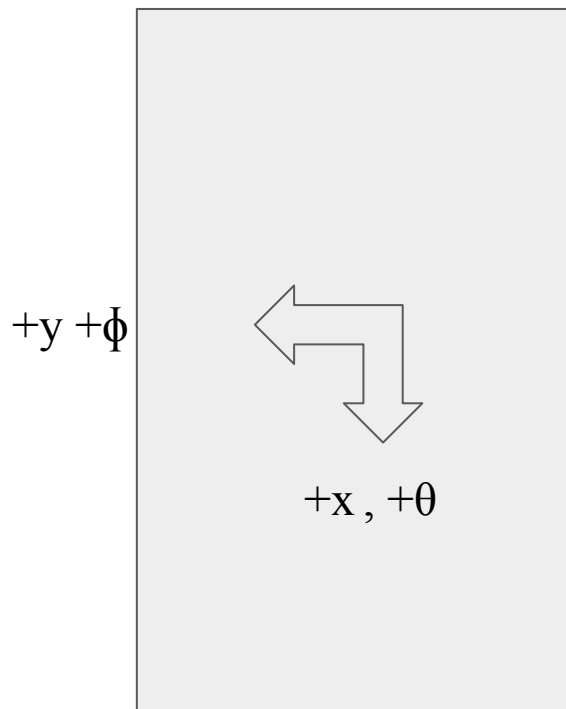
$\theta_o$  - spectrometer central angle

$\theta_{tg}$  - vertical angle,  $\phi_{tg}$  - horizontal angle

$\cos(\psi)$  correlated with  $\theta_{tg}$  and  $\sin(\psi)$  correlated  $\phi_{tg}$

# Transport Coordinate System

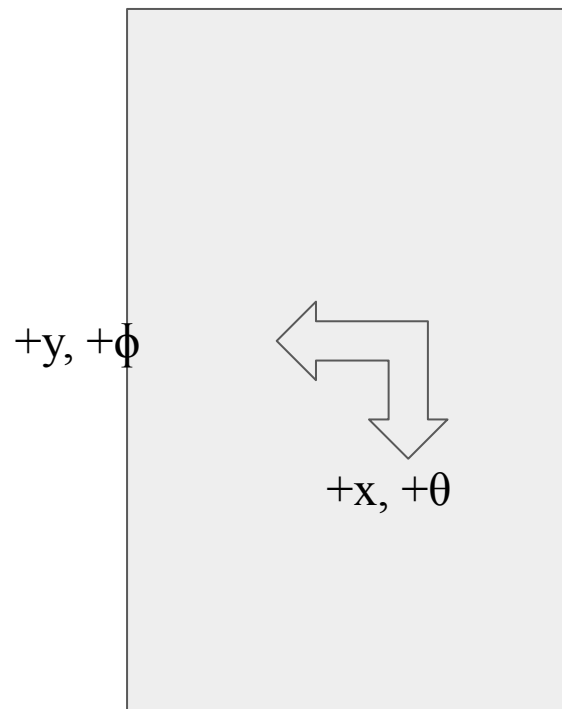
LHRS



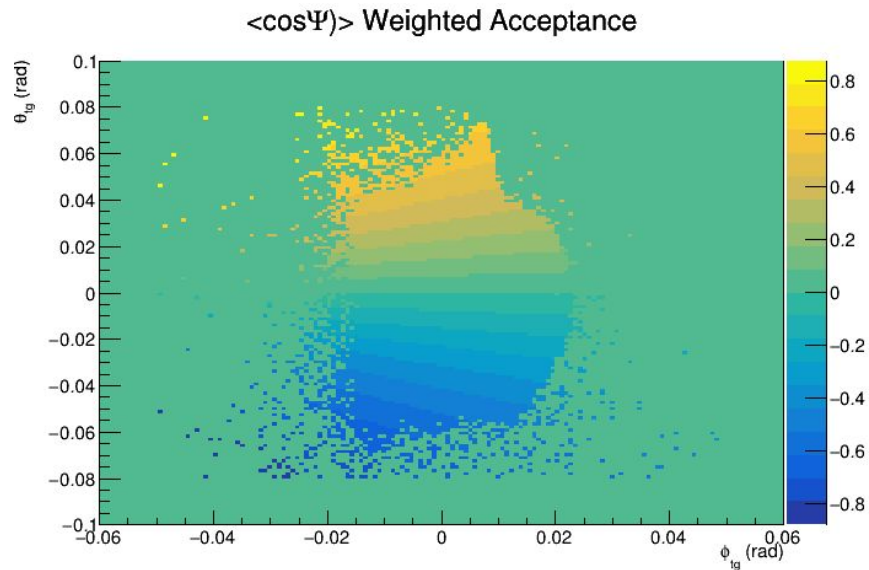
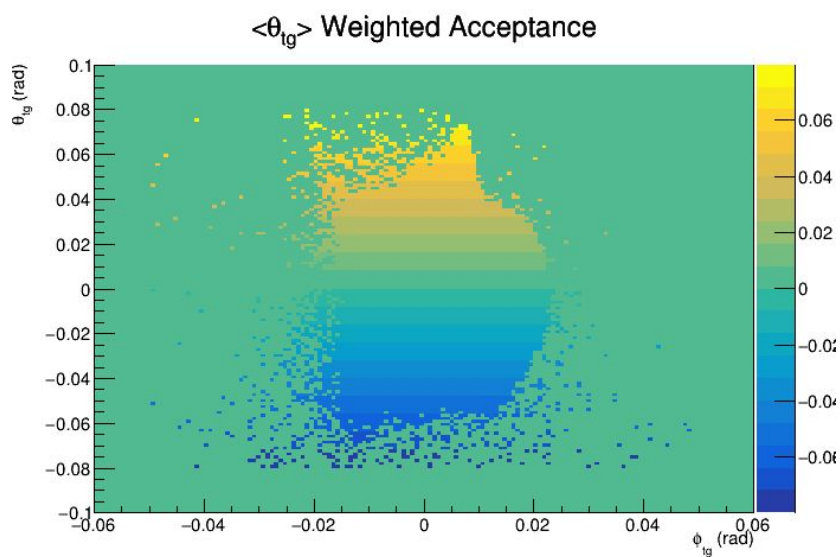
**X**

Z (beam  
direction)

RHRS



# Acceptance

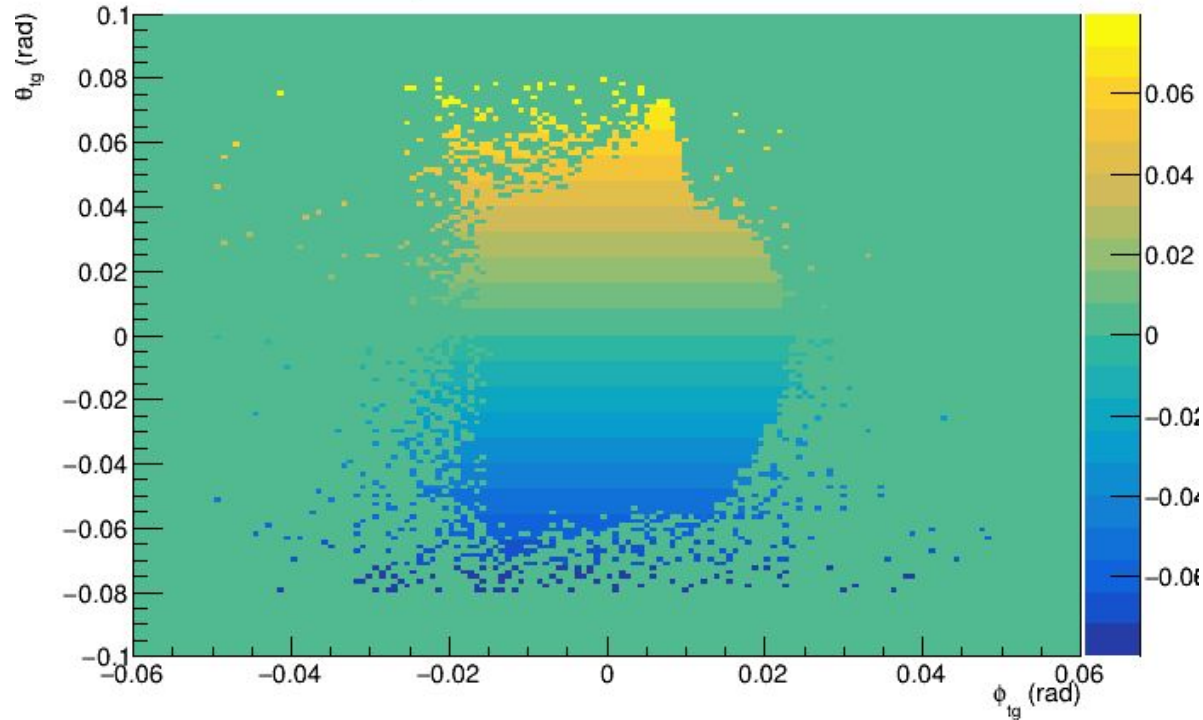


Toward negative  $\phi_{tg}$  is beam center

$\theta_{tg}$  up/down (Horizontal Pol),  $\phi_{tg}$  left/right (Vertical Pol)

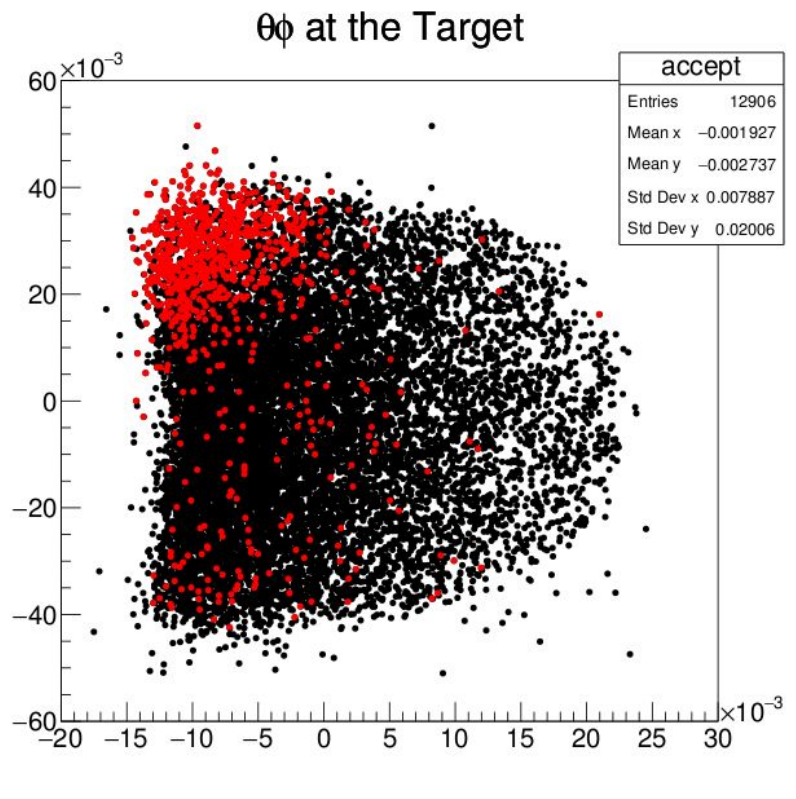
# Acceptance

$\langle \theta_{tg} \rangle$  Weighted Acceptance

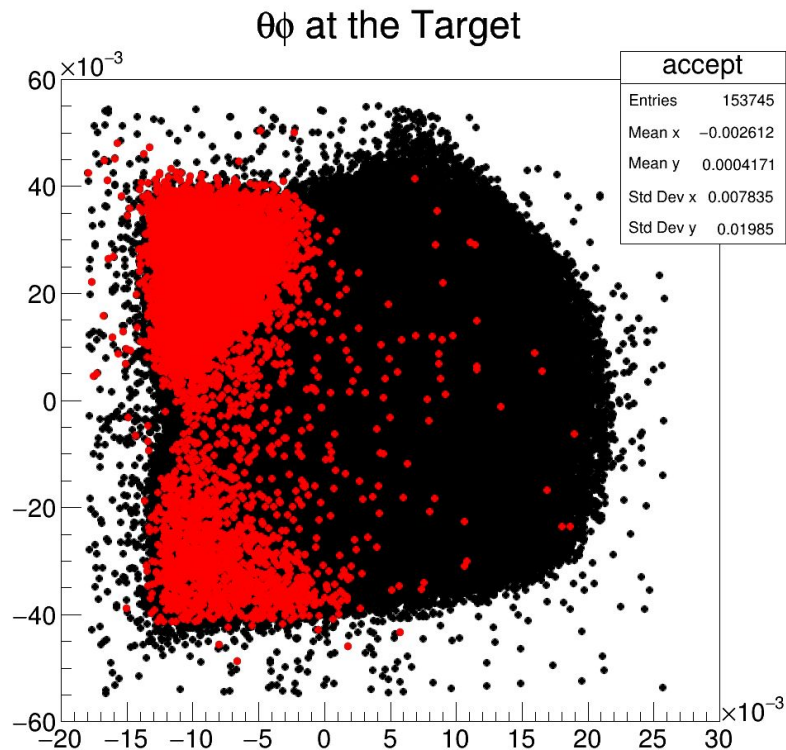


- Events at the top/bottom are  $A_T$  enhanced. Those events are more sensitive to  $A_T$  compared to events at the center of the acceptance.
- **At main detector, would like  $\langle \theta_{tg} \rangle = 0$ . Main detectors not sensitive to horizontal  $A_T$**
- $A_T$  detectors to accept events at top/bottom of acceptance.

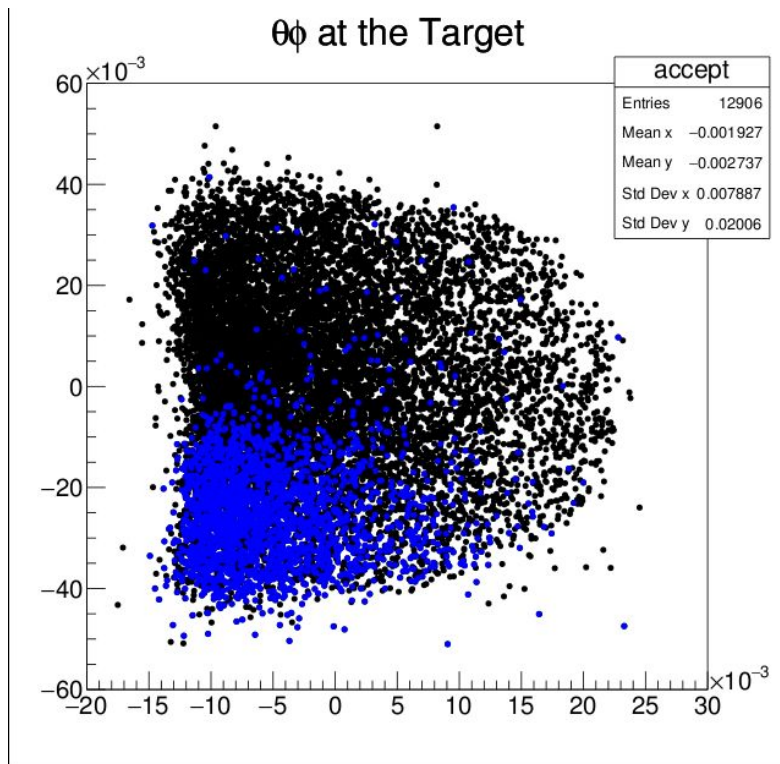
# PREX (LHRS) $A_T$ In



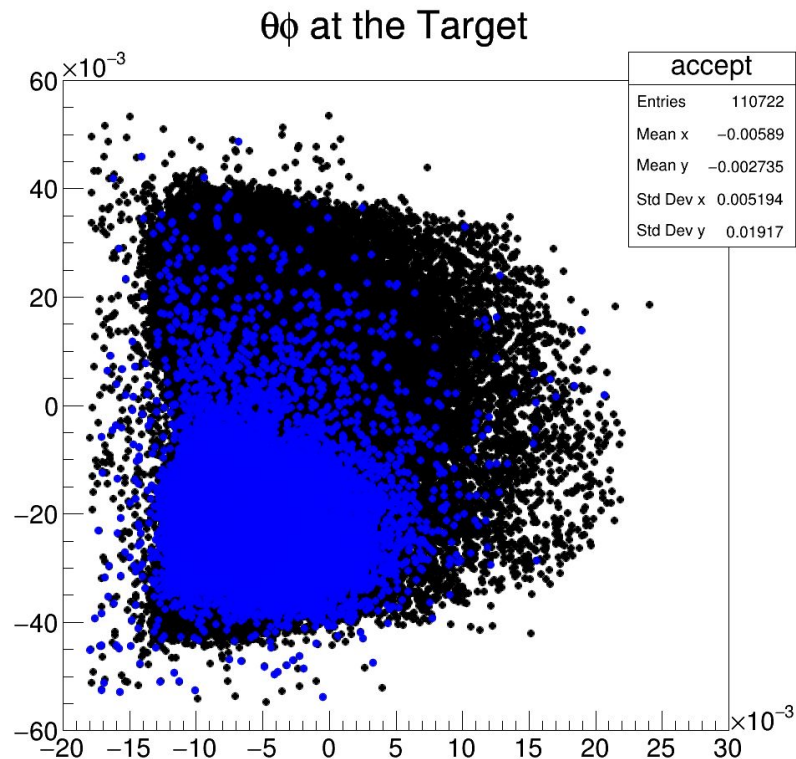
# CREX (LHRS) $A_T$ In



# PREX (LHRS) $A_T$ Out



# CREX (LHRS) $A_T$ Out





# Definitions

## Integrating Mode

$A_{m,usl}$  - reg\_asym\_usl\_mean

$$A_{PV,usl} = A_{m,usl} - A_{TV,usl} - A_{TH,usl}$$

**Same expressions for  
upstream right and  $A_T$   
detectors as well as various  
combinations**

## Counting Mode

**$A_T$  Ansatz**      $A_n = \frac{A}{Z} \hat{A}_n \sqrt{Q^2}$

$$A_{PV,usl} = P_L \hat{A}_{PV} Q_{usl}^2$$

$$A_{TV,usl} = P_V \frac{A}{Z} \hat{A}_n \sqrt{Q_{usl}^2} \langle \phi_{tg} \rangle$$

$$A_{TH,usl} = P_H \frac{A}{Z} \hat{A}_n \sqrt{Q_{usl}^2} \langle \theta_{tg} \rangle$$

**$Q^2$ ,  $\langle \theta \rangle$ ,  $\langle \phi \rangle$  are from counting**

**Hatted quantities for dimensional  
analysis**

# Vertical Transverse Polarization

Perfect spectrometer has  $\cos(\varphi) = 1$  on LHRS and  $-1$  on RHRS

Purely vertically polarized beam,  $A_{m,usl} \sim -A_{m,usr}$

$\langle \Phi_{tg} \rangle_{usl}$  and  $\langle \Phi_{tg} \rangle_{usr}$  have opposite sign

$$A_{m,usl} = P_L \hat{A}_{PV} Q_{usl}^2 + P_V \frac{A}{Z} \hat{A}_n \langle \phi_{tg} \rangle_{usl} + P_H \frac{A}{Z} \hat{A}_n \langle \theta_{tg} \rangle_{usl}$$

$$A_{m,usr} = P_L \hat{A}_{PV} Q_{usr}^2 + P_V \frac{A}{Z} \hat{A}_n \langle \phi_{tg} \rangle_{usr} + P_H \frac{A}{Z} \hat{A}_n \langle \theta_{tg} \rangle_{usr}$$

We will take the average and double difference

# Vertical Transverse Polarization

$$Q_{usavg}^2 = \frac{Q_{usl}^2 + Q_{usr}^2}{2}, \quad Q_{usdd}^2 = \frac{Q_{usl}^2 - Q_{usr}^2}{2}$$

$$A_{m,usavg} = P_L \hat{A}_{PV} Q_{usavg}^2 + P_V A_n \frac{\langle \phi_{tg} \rangle_{usl} + \langle \phi_{tg} \rangle_{usr}}{2} + P_H A_n \frac{\langle \theta_{tg} \rangle_{usl} + \langle \theta_{tg} \rangle_{usr}}{2}$$

$$A_{m,usdd} = P_L \hat{A}_{PV} Q_{usdd}^2 + P_V A_n \frac{\langle \phi_{tg} \rangle_{usl} - \langle \phi_{tg} \rangle_{usr}}{2} + P_H A_n \frac{\langle \theta_{tg} \rangle_{usl} - \langle \theta_{tg} \rangle_{usr}}{2}$$

Term sensitive to  $P_H$  is suppressed in double difference.

If  $Q_{usl}^2 \neq Q_{usr}^2$ , we must correct for it

# Vertical Transverse Polarization

$$A_{m,usavg} =_L \hat{A}_{PV} Q_{usavg}^2 + P_V A_n \frac{\langle \phi_{tg} \rangle_{usl} + \langle \phi_{tg} \rangle_{usr}}{2} + P_H A_n \frac{\langle \theta_{tg} \rangle_{usl} + \langle \theta_{tg} \rangle_{usr}}{2}$$

$$A_{m,usdd} - A_{PV,usdd} \approx P_V A_n \frac{\langle \phi_{tg} \rangle_{usl} - \langle \phi_{tg} \rangle_{usr}}{2}$$

Now we measured  $A_n$  for a purely vertically polarized beam so we have

$$A_{m,usavg}^T = P_V A_n \frac{\langle \phi_{tg} \rangle_{usl} + \langle \phi_{tg} \rangle_{usr}}{2}$$



$$\frac{A_{m,usavg}^T}{A_{m,usdd}^T} = \frac{\langle \phi_{tg} \rangle_{usl} + \langle \phi_{tg} \rangle_{usr}}{\langle \phi_{tg} \rangle_{usl} - \langle \phi_{tg} \rangle_{usr}} \equiv \xi$$

$$A_{m,usdd}^T = P_V A_n \frac{\langle \phi_{tg} \rangle_{usl} - \langle \phi_{tg} \rangle_{usr}}{2}$$

**$\xi$  is left/right apparatus asymmetry**

# Vertical Transverse Polarization

$$A_{m,usavg} =_L \hat{A}_{PV} Q_{usavg}^2 + P_V A_n \frac{\langle \phi_{tg} \rangle_{usl} + \langle \phi_{tg} \rangle_{usr}}{2} + P_H A_n \frac{\langle \theta_{tg} \rangle_{usl} + \langle \theta_{tg} \rangle_{usr}}{2}$$

$$A_{m,usdd} - A_{PV,usdd} \approx P_V A_n \frac{\langle \phi_{tg} \rangle_{usl} - \langle \phi_{tg} \rangle_{usr}}{2}$$

$$\frac{A_{m,usavg}^T}{A_{m,usdd}^T} = \frac{\langle \phi_{tg} \rangle_{usl} + \langle \phi_{tg} \rangle_{usr}}{\langle \phi_{tg} \rangle_{usl} - \langle \phi_{tg} \rangle_{usr}} \equiv \xi$$

$\xi$  measured in integrated mode. We cross check with counting data to show those measurements are within error

# Vertical Transverse Polarization Correction

**Correction to  $A_{PV}$  due to vertical transverse polarization**

$$A_{TV,usavg} = A_{m,usdd,corr} \xi, \quad A_{m,usdd,corr} = A_{m,usdd} - A_{PV,usdd}$$

Magnitude of  $P_V$  given by

$$\frac{P_V}{P} = \frac{A_{m,usdd,corr}}{A_{m,usdd}^T}$$

$$A_{m,usl} - A_{TV,usl} = A_{PV,usl} + A_{TH,usl}$$

$$A_{m,usr} - A_{TV,usr} = A_{PV,usr} + A_{TH,usr}$$

# Horizontal Transverse Polarization

Four  $A_T$  detectors to monitor transverse polarization

$$A_{m,atl1} = P_L \hat{A}_{PV} Q_{atl1}^2 + P_V A_n \langle \phi_{tg} \rangle_{atl1} + P_H A_n \langle \theta_{tg} \rangle_{atl1}$$

$$A_{m,atl2} = P_L \hat{A}_{PV} Q_{atl2}^2 + P_V A_n \langle \phi_{tg} \rangle_{atl2} + P_H A_n \langle \theta_{tg} \rangle_{atl2}$$

$$A_{m,atr1} = P_L \hat{A}_{PV} Q_{atr1}^2 + P_V A_n \langle \phi_{tg} \rangle_{atr1} + P_H A_n \langle \theta_{tg} \rangle_{atr1}$$

$$A_{m,atr2} = P_L \hat{A}_{PV} Q_{atr2}^2 + P_V A_n \langle \phi_{tg} \rangle_{atr2} + P_H A_n \langle \theta_{tg} \rangle_{atr2}$$

Different combinations can suppress the vertical transverse or vice versa

Recall that  $\langle \phi \rangle_{\text{LHRS}}$  opposite sign with  $\langle \phi \rangle_{\text{RHRS}}$

# Horizontal Transverse Polarization (LHRS)

Recall that  $\langle \phi \rangle_{\text{LHRS}}$  opposite sign with  $\langle \phi \rangle_{\text{RHRS}}$

Recall that each  $A_T$  isolates events at top/bottom of acceptance

$\langle \theta_{tg} \rangle_{\text{atl1}}$  and  $\langle \theta_{tg} \rangle_{\text{atl2}}$  have opposite signs

Consider double difference of  $A_T$ s on LHRS (same for RHRS) - vertical transverse suppressed

$$A_{m,\text{atldd}} \approx P_L \hat{A}_{PV} \frac{Q_{\text{atl1}}^2 - Q_{\text{atl2}}^2}{2} + P_H A_n \frac{\langle \theta_{tg} \rangle_{\text{atl1}} - \langle \theta_{tg} \rangle_{\text{atl2}}}{2}$$

$$A_{m,\text{uslcorr}} \equiv A_{m,\text{usl}} - A_{TV,\text{usl}} = P_L \hat{A}_{PV} Q_{\text{usl}}^2 + P_H A_n \langle \theta_{tg} \rangle_{\text{usl}}$$



# Horizontal Transverse Polarization (LHRS)

$$A_{m,atlld} \approx P_L \hat{A}_{PV} \frac{Q_{at1}^2 - Q_{at2}^2}{2} + P_H A_n \frac{\langle \theta_{tg} \rangle_{at1} - \langle \theta_{tg} \rangle_{at2}}{2}$$

$$A_{m,uslcorr} \equiv A_{m,usl} - A_{TV,usl} = P_L \hat{A}_{PV} Q_{usl}^2 + P_H A_n \langle \theta_{tg} \rangle_{usl}$$

**Subtract  $Q^2$  dependence for  $A_T$  double difference**

$$A_{m,atlld,corr} \equiv A_{m,atlld} - A_{PV,atlld} = P_H A_n \frac{\langle \theta_{tg} \rangle_{at1} - \langle \theta_{tg} \rangle_{at2}}{2}$$

$$A_{TH,usl} = P_H A_n \langle \theta_{tg} \rangle_{usl}$$

# Horizontal Transverse Polarization Correction

## Corrections due to transverse polarization

$$A_{TH,usl} = \frac{A_{m,atl\ddot{d},corr}}{\xi_{LHRS}}, \quad A_{TH,usr} = \frac{A_{m,atr\ddot{d},corr}}{\xi_{RHRS}}$$

$$\xi_{LHRS} = \frac{\langle\theta_{tg}\rangle_{atl1} - \langle\theta_{tg}\rangle_{atl2}}{2\langle\theta_{tg}\rangle_{usl}}, \quad \xi_{RHRS} = \frac{\langle\theta_{tg}\rangle_{atr1} - \langle\theta_{tg}\rangle_{atr2}}{2\langle\theta_{tg}\rangle_{usr}}$$

We have two Independent measurements of horizontal transverse polarization

One for each arm

# Q<sup>2</sup> Differences

$$A_{PV,usl} = A_{m,usl} - A_{TH,usl} - A_{TV,usl} = P_L \hat{A}_{PV} Q_{usl}^2$$

$$A_{PV,usr} = A_{m,usr} - A_{TH,usr} - A_{TV,usr} = P_L \hat{A}_{PV} Q_{usr}^2$$

Consider the average and double difference

$$A_{PV,usdd} = A_{PV,usavg} \frac{Q_{usdd}^2}{Q_{usavg}^2}$$

Correction due to Q<sup>2</sup> difference