Lagrange Multiplier Monte Carlo

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July 13, 2020

1 Introduction

The proposed Monte Carlo will produce dithering and asymmetry data that can compare regression and Lagrangian multiplier corrections for a variety of beam noise and electronic noise conditions. The basic notation is that there is one detector D, a set of BPM's B_b, dithering coils C_b, underlying beam parameters α_a, and various sources of noise.

2 The Method

The Monte Carlo generates a series of windows with the measured parameters as well as the underlying beam parameters. For the i^{th} window, the beam parameters are

$$\alpha_{ai} = \sigma_\alpha^a r_i + \Delta_a^\pm + C_b \frac{\partial \alpha_a}{\partial C_b}$$

Here r_i is a random number with zero mean and unit variance, $\Delta_a^\pm$ is the helicity correlation in α_a, C_b is the value of the current in the bth coil, and $\partial\alpha_a/\partial C_b$ gives the change in α_a due to the bth coil. The index a runs form 1-5 to cover x and y positions and energy.

The signal in the detector for the ith window is given by

$$D_i = \frac{\partial D}{\partial \alpha_a}\alpha_{ai} + \sigma_D r_i$$

For the BPM's, we have

$$B_{bi} = \frac{\partial B_b}{\partial \alpha_a}\alpha_{ai} + \sigma_b^B r_i + \beta_{ba}\sigma_a^c r_i,$$

where σ_b^B is the uncorrelated monitor noise and the term $\beta_{ba}\delta_{ai}$ allows for all possible correlated noise. Here the r_i in the $\sigma_b^B r^i$ term are all different, but the r_i in the $\beta_{ba}\sigma_a^c r_i$ term are the same for al of the monitors (but different for each window).

This data can be analyzed by the regular analysis.

3 Comments

List of constants that must be fixed for each example:

1. $\partial\alpha_a/\partial C_b$: Effect of coils on beam parameters.
2. $\partial D/\partial\alpha_a$: Sensitivity of detector to beam parameters.
3. $\partial B_b/\partial\alpha_a$: Sensitivity of the b^{th} BPM to beam parameters.
4. σ_D: Statistical noise in D.
5. $\Delta_a^{\pm}$: Helicity correlated beam difference.
6. σ_b^B: Width of BPM noise for the b^{th} monitor.
7. β_{ba}: Contribution of correlated noise a to the b^{th} Monitor.
8. σ_a^c: The a^{th} piece of BPM correlated noise in the b^{th} monitor.

The first four item should be chosen to make the Monte Carlo look like the data. The terms $\Delta_a^{\pm}$ can be made large to demonstrate systematic errors. I believe that the last two items are important for our data. In particular, if they are large, it will take many monitors to reduce the width of the detector signal.

The dithering can be done with small systematics by reducing the beam noise during the coil ramping.

4 Making the Monte Carlo look like data

To make the Monte Carlo look like data, we can start with the 4 x 4 dithering. The four downstream BPM's can be used as the beam parameters. The coefficients for the other monitors can be found by treating them as detectors and finding the position and angle sensitivity. The energy parameter can be added by estimating the dispersion for each BPM.

5 Possible Tests

1. Try to duplicate the real data. Add large systematics $\Delta_b^{\pm}$ and show that they are removed both with dithering and probably with all-monitor regression.

2. Increase the BPM noise to make regression fail. Hopefully, all dithering will work, with the smallest errors with the Lagrangian Multiplier method. This is the key test.